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Non-Markovian collision integral in Fermi systems

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Abstract

The non-Markovian collision integral is obtained on the base of the Kadanoff–Baym equations for Green functions in a form with allowance for small retardation effects. The collisional relaxation times and damping width of the giant isovector dipole resonances in nuclear matter are calculated. For an infinite Fermi liquid the dependence of the relaxation times on the collective vibration frequency and the temperature corresponds to the Landau prescription.

1. Introduction

The dissipative properties of many-body systems, specifically transport coefficients for viscosity as well as the damping of the collective excitations, are strongly governed by the interparticle collisions. In the semi-classical approach these collisions are described by a collision integral in the kinetic equation. This allows us to involve a viscosity in the motion equations similar to the phenomenological Navier–Stokes ones [1–6]. An advantage of such an approach is a conceptual clarity and a possibility to use many results from a general macroscopic physics.

For correct description of the collision relaxation rates in systems with a fast variation of the mean field, it is necessary to incorporate the memory (retardation) effects [7–16]. We investigate the form of the non-Markovian linearized collision integral with retardation using the Kadanoff–Baym equations [17, 18] for Green functions. This method enables us immediately to take into account the self-consistent mean field.

The non-Markovian collision term of the semiclassical linearized Vlasov–Landau equation was obtained with the use of the Kadanoff–Baym equations for the Green functions in [11, 14]. The kinetic equation was considered in a one-component Fermi liquid with the use of the linear approximation on deviation from overall equilibrium with a nonequilibrium distribution function $\delta f = f(\vec{p}, \vec{r}, t) - \bar{f}(\bar{\epsilon}) \propto \exp(-i\omega t)$, where $f(\vec{p}, \vec{r}, t)$ is an exact distribution function and $\bar{f}$ is an equilibrium distribution function at equilibrium single-particle energy $\bar{\epsilon}$.

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An expression for the linearized collision integral was obtained in the Born approximation on two-body collisions in the following general form:

$$J(\vec{p},\vec{r},t) = J^{(1)}(\vec{p},\vec{r},t) + J^{(2)}(\vec{p},\vec{r},t), \tag{1}$$

where the first component corresponds to variation of the distribution function and the second one is connected with the variation $\delta U = \epsilon(\vec{p},\vec{r},t) - \bar{\epsilon}$ of the mean field with $\epsilon(\vec{p},\vec{r},t)$ for the actual single-particle energy.

One can see that this relationship does not agree with the general form of the linearized Landau–Vlasov equation in a Fermi liquid in the limiting case of the states with weak time dependence of the nonequilibrium component of the distribution function, when $\partial\delta f/\partial t \to 0$. Indeed, the linearized Landau–Vlasov equation in quasi-homogeneous systems can be presented in the following form:

$$\frac{\partial\delta f}{\partial t} + \frac{p}{m}\left(\vec{\hat{p}}\cdot\frac{\partial}{\partial\vec{r}}\right)\delta\bar{f} = J, \tag{2}$$

where $\delta\bar{f}$ is a linear deviation of the exact distribution function $f(\vec{p},\vec{r},t)$ from the distribution function of the equilibrium shape $\bar{f}(\epsilon)$ evaluated with actual single-particle energy $\epsilon = \bar{\epsilon}+\delta U$,

$$\delta\bar{f} = \delta f - \frac{\mathrm{d}\bar{f}}{\mathrm{d}\bar{\epsilon}}\delta U = \delta f - \bar{f}(\epsilon). \tag{3}$$

The derivative $\partial\delta f/\partial t$ can be omitted in equation (2) for slightly time-dependent states and the collision integral should be a functional, Ψ, of the $\delta\bar{f}$,

$$J = \Psi(\delta\bar{f}) \equiv \frac{p}{m}\left(\vec{\hat{p}}\cdot\frac{\partial}{\partial\vec{r}}\right)\delta\bar{f}, \tag{4}$$

in order that the Landau–Vlasov equation is fulfilled in this case. The relationships (1) and (4) are generally in contradiction and the derivation of the form of the non-Markovian collision integral from [11, 14] can be revised.

It was pointed out in [3, 19] that collisional integrals of the type (4), $J \equiv J(\delta\bar{f})$, are the general form of the collision integrals between quasiparticles in a Fermi liquid without retardation and lead to a local equilibrium state described by distribution function $f_{le} = \bar{f}(\epsilon = \bar{\epsilon} + \delta U)$. Due to this we use in the following this terminology.

In this contribution an expression for the non-Markovian collision integral of the linearized Vlasov–Landau transport equation from [11, 14] is modified in section 2 for the case of slightly time-dependent states, i.e. in a form which allows for reaching the local equilibrium in a system.

In section 3 calculations of the relaxation times and damping width of the collective vibration in nuclear matter with the use of the collision integral with the retardation effect are presented.

2. Non-Markovian linearized collision integral within the semiclassical approach

In order to obtain the linearized Vlasov–Landau equation with the collision integral we use the mixed $\{\vec{p},\vec{r}\}$ Weyl–Wigner representation for single-particle Green functions (correlation functions) $G^{>}(1,1')$, $G^{<}(1,1')$:

$$f(\vec{p},\vec{r},t) \equiv g^{<}(t,t;\vec{p},\vec{r}) = \int \mathrm{d}\vec{r}'\,\exp\left(-\frac{\mathrm{i}}{\hbar}\vec{p}\vec{r}'\right)g^{<}\left(t,t;\vec{r}+\frac{\vec{r}'}{2},\vec{r}-\frac{\vec{r}'}{2}\right);$$
$$\vec{r}' = \vec{r}_1 - \vec{r}_{1'}, \vec{r} = \tfrac{1}{2}(\vec{r}_1 - \vec{r}_{1'}), \tag{5}$$

where $f(\vec{p},\vec{r},t)$ is one-body distribution function and Green functions are determined by [17, 18]

$$\begin{aligned} G^{>}(1,1') &= -\mathrm{i}\langle\Psi(1)\Psi^{+}(1')\rangle, \qquad t_1 > t_1', \\ G^{<}(1,1') &= \mathrm{i}\langle\Psi^{+}(1')\Psi(1)\rangle, \qquad t_1 < t_1'. \end{aligned} \tag{6}$$

Here, $\Psi^+(1)$ and $\Psi(1)$ are the operators of creation and annihilation of a fermion; the symbol 1 includes both space and time variables, namely $1 \equiv \{\vec{r}_1, t\}$ (we omit isotopic and spin variables), and the expectation values in equation (6) are calculated for the ground state or for the ensemble of initial states if the temperature of the system is not zero.

The Green functions $G^>(1, 1')$ and $G^<(1, 1')$ satisfy the Kadanoff–Baym equation

$$\begin{aligned}\left[\mathrm{i}\hbar\frac{\partial}{\partial t_1} + \frac{\hbar^2}{2m}\nabla_1^2 - U(1)\right]G^{><}(1, 1') &= I_1^{><}(1, 1') - I_2^{><}(1, 1'),\\ \left[-\mathrm{i}\hbar\frac{\partial}{\partial t_1'} + \frac{\hbar^2}{2m}\nabla_{1'}^2 - U(1')\right]G^{><}(1, 1') &= J_1^{><}(1, 1') - J_2^{><}(1, 1'),\end{aligned} \tag{7}$$

where the single-particle potentials $U(1)$, $U(1')$ are determined by the relations

$$\begin{aligned}U(1)G^{><}(1, 1') &= \int \mathrm{d}\vec{r}_2\, \Sigma_0(\vec{r}_1, \vec{r}_2, t_1)G^{><}(\vec{r}_2, t_1; 1'),\\ U(1')G^{><}(1, 1') &= \int \mathrm{d}\vec{r}_2\, G^{><}(1; \vec{r}_2, t_{1'})\Sigma_0(\vec{r}_2, \vec{r}_{1'}, t_{1'})\end{aligned} \tag{8}$$

with

$$\Sigma_0(\vec{r}_1, \vec{r}_2, t) = -\mathrm{i}\delta(\vec{r}_1 - \vec{r}_2)\int \mathrm{d}\vec{r}\, v(|\vec{r}_1 - \vec{r}|)G^<(\vec{r}, t_1; \vec{r}, t_1) + \mathrm{i}v(|\vec{r}_1 - \vec{r}_2|)G^<(\vec{r}_1, t_1; \vec{r}_2, t_1), \tag{9}$$

and $v(|\vec{r}_1 - \vec{r}_2|)$ for the two-body potential.

The functions $I_{1,2}^{><}$ and $J_{1,2}^{><}$ are correlation integrals of the form

$$\begin{aligned}I_1^{><}(1, 1') &= \frac{1}{\hbar}\int_{t_0}^{t_1} \mathrm{d}t_2\, [\Sigma^>(1, 2) - \Sigma^<(1, 2)]G^{><}(2, 1'),\\ I_2^{><}(1, 1') &= \frac{1}{\hbar}\int_{t_0}^{t_1'} \mathrm{d}t_2\, \Sigma^{><}(1, 2)[G^>(2, 1') - G^<(2, 1')],\\ J_1^{><}(1, 1') &= \frac{1}{\hbar}\int_{t_0}^{t_1} \mathrm{d}t_2\, [G^>(1, 2) - G^<(1, 2)]\Sigma^{><}(2, 1'),\\ J_2^{><}(1, 1') &= \frac{1}{\hbar}\int_{t_0}^{t_1'} \mathrm{d}t_2\, G^{><}(1, 2)[\Sigma^>(2, 1') - \Sigma^<(2, 1')].\end{aligned} \tag{10}$$

These correlation integrals have the general form with allowance for the retardation effect. It is assumed as usual that interaction between particles starts at the time $t_0 = -\infty$.

In order to obtain the Landau–Vlasov equation the following suggestions are adopted [11, 14]:

(A) the Born approximation for two-body scattering;
(B) the time variation of the nonequilibrium distribution function $\delta f \propto \exp(-\mathrm{i}\omega t)$ is taken as a periodic one with real frequency during the whole time interval $(-\infty \leqslant t' \leqslant t)$;
(C) the linear approximation on the deviation of one-body Green functions from their equilibrium values is used;
(D) the Fermi system is considered as a quasi-homogeneous one in coordinate space.

The linearized Vlasov–Landau equation has the form (2) and can be presented as [11, 14]

$$\frac{\partial}{\partial t}f(\vec{p}, \vec{r}, t) + \{\varepsilon, f\} = J(\vec{p}, \vec{r}, t). \tag{11}$$

Here,

$$\{\varepsilon, f\} \equiv \frac{\partial}{\partial \vec{p}}\varepsilon \cdot \frac{\partial}{\partial \vec{r}}f - \frac{\partial}{\partial \vec{r}}\varepsilon \cdot \frac{\partial}{\partial \vec{p}}f$$

are the Poisson brackets and

$$\epsilon(\vec{p},\vec{r},t) = \frac{p^2}{2m} + U(\vec{p},\vec{r},t) \tag{12}$$

is the classical energy of a particle in the mean field $U(\vec{p},\vec{r},t) = \bar{\epsilon}+\delta U(\vec{p},\vec{r},t)$, where δU can be expressed in terms of the Landau interaction amplitude $F(\vec{p},\vec{p}')$ for the two-body collision scattering matrix:

$$\delta U = \frac{g}{N_F}\int \frac{\mathrm{d}\vec{p}'}{(2\pi\hbar)^3} F(\vec{p},\vec{p}')\delta f(\vec{p}',\vec{r};t), \tag{13}$$

where $N_F = 2p_F m^*/(g\pi^2\hbar^3)$, p_F is the Fermi momentum, m^* is the effective mass of the nucleon and g is the spin degeneracy factor.

The linearized collision integral has the form (1) with (see equations (42), (43) and (45), (46) of [14] for details)

$$J^{(j)}(\vec{p},\vec{r},t) = 2\int \frac{\mathrm{d}\vec{p}_2\,\mathrm{d}\vec{p}_3\,\mathrm{d}\vec{p}_4}{(2\pi\hbar)^6} W(\vec{p}_1,\vec{p}_2,\vec{p}_3,\vec{p}_4)\delta(\Delta\vec{p})B^{(j)}(\vec{p},\vec{r},t). \tag{14}$$

Here, $W(\{\vec{p}_i\}) = (\mathrm{d}\sigma/\mathrm{d}\Omega)4(2\pi\hbar)^3/m^2$ is the probability of two-body collisions with the initial momenta $\vec{p}_1 = \vec{p}$, $\vec{p}_2$ and final ones $\vec{p}_3$, $\vec{p}_4$; $\mathrm{d}\sigma/\mathrm{d}\Omega$ is the in-medium differential cross-section (in the Born approximation);

$$\begin{aligned} B^{(1)}(\vec{p},\vec{r},t) &= \sum_{k=1}^{4}\delta f_k(t)\frac{\partial Q(\{\bar{f}_j\})}{\partial \bar{f}_k}[\delta_+(\Delta\bar{\epsilon}+\hbar\omega)+\delta_-(\Delta\bar{\epsilon}-\hbar\omega)], \\ B^{(2)}(\vec{p},\vec{r},t) &= Q(\{\bar{f}_j\})\frac{\Delta(\delta U(t))}{\hbar\omega}\{[\delta_+(\Delta\bar{\epsilon}+\hbar\omega)-\delta_+(\Delta\bar{\epsilon})]-[\delta_-(\Delta\bar{\epsilon}-\hbar\omega)-\delta_-(\Delta\bar{\epsilon})]\}, \end{aligned} \tag{15}$$

where $\bar{f}_k \equiv \bar{f}(\vec{p}_k,\vec{r})$; $\partial Q(\{\bar{f}_j\})/\partial\bar{f}_k$ are the derivatives of the Pauli blocking factor Q with respect to $\bar{f}_k$,

$$Q(\{\bar{f}_j\}) = (1-\bar{f}_1)(1-\bar{f}_2)\bar{f}_3\bar{f}_4 - \bar{f}_1\bar{f}_2(1-\bar{f}_3)(1-\bar{f}_4). \tag{16}$$

The $\bar{\epsilon}_i = \bar{\epsilon}(\vec{p}_i,\vec{r})$ and δU_j are the equilibrium single-particle energy and the variation of the mean field for a particle with momentum $\vec{p}_i$ respectively; $\Delta\bar{\epsilon} = \bar{\epsilon}_1+\bar{\epsilon}_2-\bar{\epsilon}_3-\bar{\epsilon}_4$, $\Delta(\delta U) \equiv \delta U_1+\delta U_2-\delta U_3-\delta U_4$, $\Delta\vec{p} = \vec{p}_1+\vec{p}_2-\vec{p}_3-\vec{p}_4$. The equilibrium distribution function $\bar{f}_k \equiv \bar{f}(\vec{p}_k,\vec{r})$ depends on the equilibrium single-particle energy $\bar{\epsilon}_k \equiv \bar{\epsilon}(\vec{p}_k,\vec{r})$: $\bar{f}_k = \bar{f}(\bar{\epsilon}_k)$. It equals the Fermi function evaluated at temperature T, $\bar{f}(\bar{\epsilon}_k) = 1/[1+\exp((\bar{\epsilon}_k-\mu)/T)]$.

Note that the generalized functions δ_+, δ_- appearing in equation (15) include also an integral contribution,

$$\delta_+(x) = \frac{1}{2\pi}\int_{-\infty}^{0}\mathrm{d}\tau\,\mathrm{e}^{-\mathrm{i}x\tau} = \frac{\mathrm{i}}{2\pi}\frac{1}{x+\mathrm{i}0} = \frac{1}{2}\delta(x) - \frac{1}{2\pi\mathrm{i}}\mathcal{P}\left(\frac{1}{x}\right), \qquad \delta_-(x) = \delta_+^*(x), \tag{17}$$

where $\delta(x)$ is the delta function and the symbol $\mathcal{P}$ denotes the principal value of the integral contribution. The integral terms of the $\delta_\pm$, corresponding to virtual transitions, are usually rejected in the J because they are assumed to be included by renormalizing the interactions between particles [8]. This corresponds to substitution of the $\delta(x)/2$ for $\delta_\pm$ in equation (15), i.e., to taking into account real transitions with conservation of energy. We will consider below only these transitions.

Now we will modify the expression for quantity $B^{(1)}$. We present the nonequilibrium component δf of the distribution function in the form

$$\delta f(\vec{p}_j,\vec{r},t) = -\nu(\vec{p}_j,\vec{r},t)\frac{\partial\bar{f}(\bar{\epsilon}_j)}{\partial\bar{\epsilon}_j}. \tag{18}$$

Then the quantity $B^{(1)}$ can be written as

$$B^{(1)}(\vec{p},\vec{r},t) = -\frac{1}{2}\sum_{k=1}^{4}\nu_k\frac{\partial Q(\{\bar{f}_j\})}{\partial\bar{\epsilon}_k}[\delta(\Delta\bar{\epsilon}+\hbar\omega)+\delta(\Delta\bar{\epsilon}-\hbar\omega)]$$
$$= \frac{1}{2}\Delta\nu Q(\{\bar{f}_j\})\frac{\partial}{\partial\hbar\omega}[\delta(\Delta\bar{\epsilon}+\hbar\omega)+\delta(\Delta\bar{\epsilon}-\hbar\omega)] - \delta B^{(1)}, \tag{19}$$

where $\Delta\nu \equiv \nu_1+\nu_2-\nu_3-\nu_4$, $\nu_k = \nu(\vec{p}_k,\vec{r},t)$ and

$$\delta B^{(1)} = \frac{1}{2}\sum_{k=1}^{4}\frac{\partial}{\partial\bar{\epsilon}_k}\{\nu_k Q(\{\bar{f}_j\})[\delta(\Delta\bar{\epsilon}+\hbar\omega)+\delta(\Delta\bar{\epsilon}-\hbar\omega)]\}$$
$$+\frac{1}{2}\sum_{k=1}^{4}Q(\{\bar{f}_j\})[\delta(\Delta\bar{\epsilon}+\hbar\omega)+\delta(\Delta\bar{\epsilon}-\hbar\omega)]\frac{\partial\nu_k}{\partial\bar{\epsilon}_k}. \tag{20}$$

The first component in equation (20) determines a probability flux of colliding particles which is connected with a possibility of variation of the energy $\bar{\epsilon}_k$ when the values of other energies ($\bar{\epsilon}_{j\neq k}$ and $\hbar\omega$) are fixed. This term should be equal to zero because of fixing the total energy in initial or final states and therefore it does not contribute to the total number of collisions $\mathcal{N}$:

$$\mathcal{N}(\hat{p}) \equiv \int_0^\infty \mathrm{d}\epsilon\, J(\hat{p},\epsilon), \qquad \hat{p}\equiv\vec{p}/p. \tag{21}$$

A relative dynamical component ν_k of the distribution function is slowly dependent on energy and it can be considered (at least for low temperatures $T \ll \epsilon_F$) as a function of the momentum direction rather than of the momentum: $\nu_k \equiv \nu(\vec{p}_k,\vec{r},t) = \nu(\hat{p}_k,\epsilon_F,\vec{r},t)$. Therefore the second component in equation (20) is also negligible and the term $\delta B^{(1)}$ should be omitted from equation (19), $\delta B^{(1)} = 0$. The expressions for $B^{(j)}$ take the form

$$B^{(1)}(\vec{p},\vec{r},t) = \frac{1}{2}\Delta\nu Q(\{\bar{f}_j\})\frac{\partial}{\partial\hbar\omega}[\delta(\Delta\bar{\epsilon}+\hbar\omega)+\delta(\Delta\bar{\epsilon}-\hbar\omega)],$$
$$B^{(2)}(\vec{p},\vec{r},t) = \frac{1}{2}Q(\{\bar{f}_j\})\frac{\Delta(\delta U(t))}{\hbar\omega}\{[\delta(\Delta\bar{\epsilon}+\hbar\omega)-\delta(\Delta\bar{\epsilon})]-[\delta(\Delta\bar{\epsilon}-\hbar\omega)-\delta(\Delta\bar{\epsilon})]\}. \tag{22}$$

The shift in energy $\Delta\bar{\epsilon}$ by $\hbar\omega$ in the arguments of the δ-functions of the expressions (1), (14), (22) for the collision integral agrees with the interpretation of the collisions in the presence of the collective excitations proposed by Landau [20]. According to this interpretation, high-frequency oscillations in a Fermi liquid can be considered as phonons, that are absorbed and created at the two-particle collisions.

As discussed in the introduction, the form of expressions (1), (14), (22) for the collision integral is not correct in the general case and an additional modification of these expressions is needed. The incorrectness results from approximations which were made at the kinetic equation derivation.

Because of assumptions both on the undamped behaviour of the distribution function throughout the time interval ($-\infty \leqslant t' \leqslant t \to \infty$) and on the small magnitude of two-body interaction, one can expect an overestimation of retardation effects in the foregoing expression for the collision integral. This means that from a physical point of view this relationship should be fulfilled only in the case of small ω.

Therefore, we replace the derivatives of the form $\partial\delta(\Delta\bar{\epsilon}+\hbar\omega)/\partial\hbar\omega$ and $\partial\delta(\Delta\bar{\epsilon}-\hbar\omega)/\partial\hbar\omega$ in $B^{(1)}$ of the equation (22) by the finite differences $(\delta(\Delta\bar{\epsilon}+\hbar\omega)-\delta(\Delta\bar{\epsilon}))/\hbar\omega$ and $(\delta(\Delta\bar{\epsilon})-\delta(\Delta\bar{\epsilon}-\hbar\omega))/\hbar\omega$, respectively. Then we combine the resulting expression with

the contribution $B^{(2)}$ arising from the mean-field variation and finally obtain the linearized collision integral for the Fermi liquid in the following form:

$$J(\vec{p},\vec{r},t)=\int\frac{\mathrm{d}\vec{p}_2\,\mathrm{d}\vec{p}_3\,\mathrm{d}\vec{p}_4}{(2\pi\hbar)^6}W(\{\vec{p}_i\})\delta(\Delta\vec{p})\Delta\chi\,Q\frac{\delta(\Delta\epsilon+\hbar\omega)-\delta(\Delta\epsilon-\hbar\omega)}{\hbar\omega}. \tag{23}$$

Here, $\Delta\chi\equiv\chi_1+\chi_2-\chi_3-\chi_4$; $\chi_k=\chi(\vec{p}_k,\vec{r},t)$ is a function determining the relative deviation of the distribution function from the local equilibrium state, $\delta\bar{f}$, (3):

$$\delta\bar{f}=\delta f-\frac{\mathrm{d}\bar{f}}{\mathrm{d}\bar{\epsilon}}\delta U=f(\vec{p},\vec{r},t)-f_{le}=-\chi\frac{\mathrm{d}\bar{f}}{\mathrm{d}\bar{\epsilon}},\qquad \chi=\nu+\delta U. \tag{24}$$

With the use of the algebraic relation [4]

$$\left[(1-\bar{f}_1)(1-\bar{f}_2)\bar{f}_3\bar{f}_4-\bar{f}_1\bar{f}_2(1-\bar{f}_3)(1-\bar{f}_4)\exp\left(\frac{\mp\hbar\omega}{T}\right)\right]\delta(\Delta\epsilon\pm\hbar\omega)=0, \tag{25}$$

equation (23) can be presented as

$$\begin{aligned}J(\vec{p},\vec{r},t)=&\int\frac{\mathrm{d}\vec{p}_2\,\mathrm{d}\vec{p}_3\,\mathrm{d}\vec{p}_4}{(2\pi\hbar)^6}W(\{\vec{p}_i\})\delta(\Delta\vec{p})\Delta\chi\,\bar{f}_1\bar{f}_2(1-\bar{f}_3)(1-\bar{f}_4)\\&\times[\Phi(\hbar\omega,T)-\Phi(-\hbar\omega,T)],\end{aligned} \tag{26}$$

where $\Phi(\hbar\omega,T)=\delta(\Delta\epsilon+\hbar\omega)[\exp(-\hbar\omega/T)-1]/2\hbar\omega$.

The collision integral of the form (23) or (26) depends on the variation $\delta\bar{f}$, $J\equiv J(\delta\bar{f})$. It was mentioned in the introduction that this behaviour is in line with general properties of the Vlasov–Landau equation in the Fermi liquid [3, 4] at $\partial\delta f/\partial t=0$. The collision integral provides a driving of the distribution function towards its local equilibrium value. This behaviour is in line with the general properties of the Vlasov–Landau equation in the Fermi-liquid [3, 4] at $\partial\delta f/\partial t=0$. The expressions (23), (26) depend only on the occupation probability $\mathcal{P}_{2p2h}\equiv\bar{f}_1\bar{f}_2(1-\bar{f}_3)(1-\bar{f}_4)$ of the 2p–2h states in the phase space. This fact leads to interpretation of the collision damping with a linearized collision term (26) as a relaxation process due to the coupling of one-particle and one-hole states to more complicated 2p–2h configurations.

The form of the collision integral (26) in the Markovian limit ($\omega\to0$) coincides with the standard expression for the collision integral in a Fermi liquid without retardation effects [3, 4] because in this case the term in square brackets of equation (26) tends to the value $-\delta(\Delta\epsilon)/T$.

Equation (23) for some special explicit form of the quantity χ_j was first used in [9, 21, 22]. The derivation of the collision integral (23) is performed in [15] within the framework of the extended time-dependent Hartree–Fock model. The expressions for the distortion functions χ_j corresponding to a perturbation approach on the collision term and including the amplitudes of the random phase approximation were used in this method.

Note that the expression for the collision integral in the two-component Fermi system is obtained from equation (26) in the same manner as done in [23] under the assumption that the chemical potentials and the equilibrium distribution functions are the same for protons and neutrons.

3. Calculations of the relaxation times and nuclear matter viscosity

The collision integral can be used to calculate collisional relaxation times governing the dissipative properties of different physical quantities [2, 4, 9–12, 24–26]. Below we calculate relaxation times, $\tau_\ell^{(\pm)}$, of collective vibrations in two-component nuclear matter consisting

of neutron and protons subsystems. These collective relaxation times are determined by interparticle collisions within the distorted layers of the Fermi surface with multipolarity ℓ:

$$\frac{1}{\tau_\ell^{(\pm)}} \equiv \int_0^\infty \mathrm{d}\epsilon_1 \int \mathrm{d}\Omega_p \, J_c^{(\pm)}(\hat{p},\epsilon_1) Y_{\ell 0}(\hat{p}) \Big/ \int_0^\infty \mathrm{d}\epsilon_1 \int \mathrm{d}\Omega_p \, \delta f^{(\pm)} Y_{\ell 0}(\hat{p}), \tag{27}$$

where $Y_{\ell m}(\hat{p})$ is the spherical harmonic function and $J^{(\pm)}(\hat{p},\epsilon)$ are the linear combinations of the collision integrals for protons J_p and neutrons J_n in nuclear matter: $J^{(\pm)} = (J_p \pm J_n)/2$; $\delta f^{(\pm)} = (\delta f_p \pm \delta f_n)/2$. These times are proportional to the relaxation times $\tau_c^{(\pm)}$ defining the damping widths $\Gamma_c^{(\pm)}(L)$ of the isoscalar (+) and the isovector (−) vibrations with frequency ω [12, 23, 26, 27] in the regime of rare collisions with $\omega\tau_c^{(\pm)} \gg 1$ in the Fermi liquid. In particular, the collisional damping widths of giant resonances with dipole ($L = 1$) and quadrupole ($L = 2$) multipolarities resemble the widths in the relaxation rate approach

$$\Gamma_c^{(\pm)}(L) = \hbar/\tau_c^{(\pm)}(L), \qquad \tau_c^{(-)}(L=1) = \tau_{\ell=1}^{(-)}, \qquad \tau_c^{(+)}(L=2) = \tau_{\ell=2}^{(+)}, \tag{28}$$

in the case when the nuclear fluid dynamical model with relaxation is used [23, 26]. The collisional damping width [12] of zero sound in the Fermi liquid with its relative velocity $S_r \simeq 1$ is also given by equation (28) but with the use of $\tau_{\ell\to\infty}^{(+)} \propto \tau_2^{(+)}$ for $\tau_c^{(\pm)}(L)$.

The time $\tau_{\ell=2}^{(+)}$ at $\omega = 0$ is the thermal relaxation time determining the viscosity coefficient of the Fermi liquid [25].

The analytical expressions for collective relaxation times of the damping of the collective vibrations with frequency ω have the following general form in low-temperature and low-frequency limits ($T, \hbar\omega \ll \epsilon_F$) [12, 23, 28]

$$\frac{\hbar}{\tau_\ell^{(\pm)}} = \frac{1}{\alpha_\ell^{(\pm)}}\{(\hbar\omega)^2 + (2\pi T)^2\}, \qquad \frac{1}{\alpha_\ell^{(\pm)}} = \frac{2m}{3\pi\hbar^2}[\langle \sigma'_{av}\Phi_\ell^{(+)}\rangle + \langle \sigma'_{pn}\Phi_\ell^{(\pm)}\rangle], \tag{29}$$

where $\sigma'_{av} = (\sigma'_{nn} + \sigma'_{pp})/2$; $\sigma'_{jj'} \equiv \mathrm{d}\sigma_{jj'}/\mathrm{d}\Omega$ is the in-medium differential cross-section for scattering of the nucleons j and j' (here, $j = n$ or p, and similarly $j' = p$ or n). The symbol $\langle\cdots\rangle$ in equation (29) denotes averaging over angles of the relative momenta of the colliding particles,

$$\langle(\cdots)\rangle = \frac{1}{\pi}\int_0^\pi \mathrm{d}\phi \sin(\phi/2) \int_0^\pi \mathrm{d}\theta \, (\cdots). \tag{30}$$

The functions $\Phi_\ell^{(\pm)}$ define the angular constraint on nucleon scattering within the distorted layers of the Fermi surface with multipolarity ℓ:

$$\Phi_\ell^{(\pm)} = 1 \pm P_\ell(\cos\phi) - P_\ell((\hat{p}_3\hat{p}_1)) \mp P_\ell((\hat{p}_4\hat{p}_1)), \tag{31}$$

where the scalar products $(\hat{p}_3\hat{p}_1)$ and $(\hat{p}_4\hat{p}_1)$ are given as

$$\begin{aligned} (\hat{p}_3\hat{p}_1) &= \cos^2(\phi/2) + \sin^2(\phi/2)\cos\theta, \\ (\hat{p}_4\hat{p}_1) &= \cos^2(\phi/2) - \sin^2(\phi/2)\cos\theta. \end{aligned} \tag{32}$$

Due to the momentum conservation and conditions $p_i = p_F$, the angle θ agrees with the scattering angle in the centre-of-mass reference frame of two nucleons. The angle ϕ defines the magnitudes of the relative momenta $\vec{k}_i = (\vec{p}_2 - \vec{p}_1)/2$ and $\vec{k}_f = (\vec{p}_4 - \vec{p}_3)/2$ before and after collision, respectively. The value of total momentum, $\vec{P} = \vec{p}_1 + \vec{p}_2$, also depends on the magnitude of ϕ. We have

$$\vec{k}_i\vec{k}_f = k^2\cos\theta, \qquad k^2 = k_i^2 = k_f^2 = p_F^2\sin^2(\phi/2), \qquad \vec{P}^2 = 4p_F^2\cos^2(\phi/2). \tag{33}$$

Therefore the relative kinetic energy E_{rel} of two nucleons as well as the energy of the centre of mass motion E_{cm} are dependent on angle ϕ

$$E_{rel} = k^2/m = 2\epsilon_F\sin^2(\phi/2), \qquad E_{cm} = P^2/2m = 2\epsilon_F\cos^2(\phi/2) \tag{34}$$

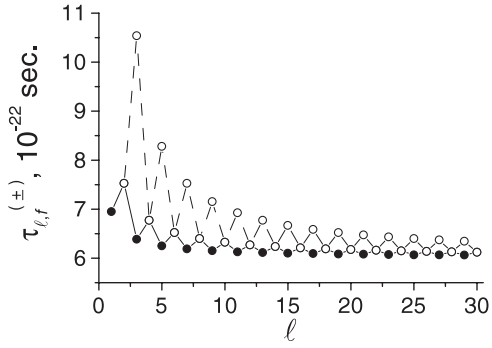

Figure 1. The relaxation times $\tau_{\ell,f}^{(\pm)}$ versus multipolarity ℓ in a cold nucleus ^{208}Pb with free space cross-section.

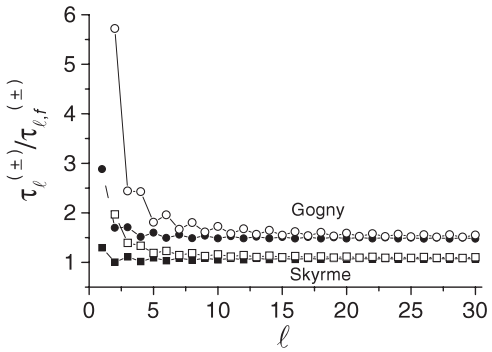

Figure 2. The relative relaxation times $\tau_{\ell}^{(\pm)}/\tau_{\ell,f}^{(\pm)}$ versus multipolarity ℓ for different cross-section parametrizations.

and only the total energy $E_{tot} = E_{rel} + E_{cm}$ remains fixed, $E_{tot} = 2\epsilon_F$. Therefore the in-medium differential cross-sections $\sigma'_{j,m}$ of two nucleons scattering depend on the relative momenta $\vec{k}_i$ and $\vec{k}_f$ at fixed total energy rather then at fixed relative kinetic energy E_{rel}, because the magnitude of E_{rel} changes with angle ϕ between colliding particles. The transfer momenta $\vec{q} = \vec{k}_i - \vec{k}_f = \vec{p}_3 - \vec{p}_1$ and $\vec{q}' = -(\vec{k}_i + \vec{k}_f) = \vec{p}_1 - \vec{p}_4$ for scattering due to direct and exchange interactions respectively are also functions of ϕ and θ: $q = 2k(\phi)\sin(\theta/2)$ and $q' = 2k(\phi)\cos(\theta/2)$.

Now we estimate the collisional relaxation times in the case of the isotropic scattering with the angle-integrated cross-sections $\sigma_{jj'}$ independent of energy. Performing angular integration in (29) with the use of equations (30) and (31) we find that $1/\tau^{(\pm)}_{\ell<\ell_0^{(\pm)}} = 0$ and

$$\frac{\hbar}{\tau_\ell^{(\pm)}} = \frac{1}{\alpha_\ell^{(\pm)}}[(\hbar\omega/2\pi)^2 + T^2], \qquad \frac{1}{\alpha_\ell^{(\pm)}} = \frac{8m}{3\hbar^2}[c_\ell\sigma_{av} + d_\ell^{(\pm)}\sigma_{np}],$$
$$c_\ell = 1 - \frac{2-(-1)^\ell}{2\ell+1}, \qquad d_\ell^{(-)} = \frac{1-(-1)^\ell}{2\ell+1}, \qquad d_\ell^{(+)} = d_{\ell=0}^{(-)} = c_{\ell=0} = c_{\ell=1} = 0, \tag{35}$$

where $\sigma_{av} = [\sigma_{pp} + \sigma_{nn} + 2\sigma_{np}]/4$ is the in-medium spin–isospin averaged nucleon–nucleon cross-section. The magnitude of the in-medium cross-section $\sigma_{jj'}$ is usually taken as proportional to the value of the free space cross-section $\sigma_{jj'}^{(f)}$ with a factor $F = \sigma_{jj'}/\sigma_{jj'}^{(f)}$,

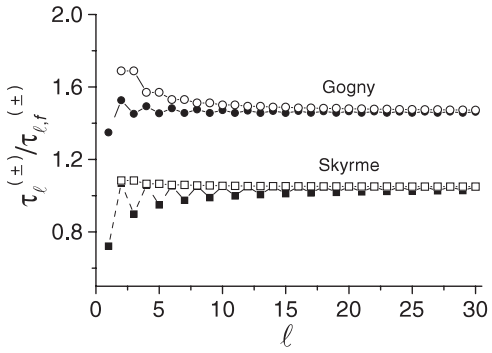

Figure 3. The relative relaxation times $\tau_\ell^{(\pm)}/\tau_{\ell,f}^{(\pm)}$ versus multipolarity ℓ for different cross-section parametrizations calculated on the Fermi surface.

so that the parameter $\alpha_\ell^{(\pm)}$ can be rewritten in the form

$$\alpha_\ell^{(\pm)} = \tilde{\alpha}_\ell^{(\pm)}/F, \qquad \tilde{\alpha}_\ell^{(\pm)} = 4.18/[c_\ell + 1.3d_\ell^{(\pm)}],\ \text{MeV}. \tag{36}$$

Here, the values $\sigma_{av}^{(f)} = 3.75$ fm^2 and $\sigma_{np}^{(f)} = 5$ fm^2 are adopted [9, 27]; they correspond to the free space cross-sections near the Fermi energy. The dependence on ℓ for relaxation times $\tau_\ell^{(\pm)} \equiv \tau_{\ell,f}^{(\pm)}$ with the use of free space cross-sections is shown in figure 1; $\hbar\omega = 13.43$ MeV. Figures 2 and 3 describe relative relaxation times versus multipolarity ℓ for cross-section parametrizations using Gogny and Skyrme effective two-body forces with parameters from [15, 30]. The relative relaxation times presented in figure 3 were calculated using cross-section on Fermi surface. Solid and dashed curves connect the values which correspond to isoscalar and isovector modes of vibrations respectively.

The magnitudes of the relaxation times are different for isoscalar and isovector modes of vibrations and they are dependent on the multipolarity ℓ. The collisional relaxation times vary rather slowly with collective motion mode at isotropic scattering with energy independent free cross-sections. As seen from figure 3, the relaxation times calculated with the effective interaction between nucleons are greater than those with cross-section in free space. This means that the in-medium cross-section between nucleons in nuclear matter is smaller than in free space (≈20% for Skyrme forces and ≈60% for Gogny interaction). The relaxation times $\tau_\ell^{(\pm)}$ depend on frequency ω due to the memory effects in the collision integral. The temperature dependence arises from smearing out the equilibrium distribution function near the Fermi momentum in heated nuclei. The collisional rates $1/\tau_\ell^{(\pm)}$ are quadratic both in temperature and in frequency with the same relationship between the components much as in the zero sound attenuation factor of a heated Fermi liquid within the Landau prescription [9, 15, 20, 26].

According to the response function approach the damping width of the giant isovector dipole resonance (GDR) at temperature T is presented in the form [29]

$$\Gamma(T) = 2q\gamma \frac{E_r^2 + E_0^2}{(E_r^2 - E_0^2)^2 + (2\gamma E_r)^2}, \tag{37}$$

where γ is determined by the relaxation time $\tau_c(\hbar\omega = E_r, T)$,

$$\gamma = \frac{\hbar}{\tau_c(\hbar\omega = E_r, T)}, \tag{38}$$

E_r is the energy of the giant dipole resonance and $E_0 = 41A^{-1/3}$ MeV. Here, we determine the quantity q from the equality of the GDR width in cold nuclei with the corresponding

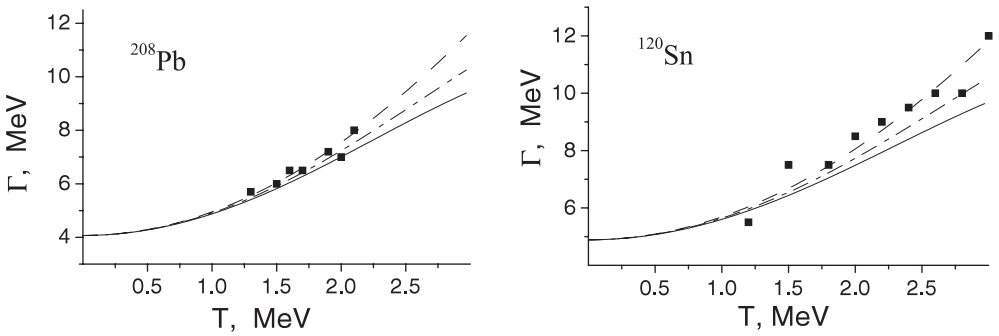

Figure 4. The temperature dependence of the GDR width according to equation (38) in nuclei ^{208}Pb and ^{120}Sn: dashed curves—calculations with Skyrme forces; dash–dot curves—Gogny interaction; solid curves—calculations with free space cross-section for two-nucleon collisions.

experimental value Γ_{exp}: $\Gamma(T=0)=\Gamma_{exp}$. The temperature dependences of the GDR width according to equation (38) in atomic nuclei ^{208}Pb and ^{120}Sn are shown in figure 4. Experimental data are taken from [31, 32] and they are indicated by points. The relaxation time τ_c is taken as equal to $\tau^{(-)}_{\ell=1}$: dashed curves in figure 4 correspond to calculations with Skyrme forces; dash–dot curves—Gogny interaction; solid curves—calculations with free space cross-section for two-nucleon collisions. The temperature behaviour of the damping width is in rather close agreement with those of experimental data.

It is seen from figure 4 that in the rare collision regime the dependence of the GDR widths on the collective vibration frequency and the temperature has the following form: $\Gamma \propto (\hbar\omega)^2 + 4(\pi T)^2$, which corresponds to the Landau prescription [20, 33, 34].

The relaxation times vary rather slowly with multipolarity of the Fermi surface distortions governed by collective motion and two-body collisions. This gives us the possibility to use approximately the relaxation time ansatz for the collision integral.

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